

Compact Star Matter: EoS with New Scaling Law

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In this paper we present a simple discussion on the properties of compact stars using an EoS obtained in effective field theory anchored on scale and hidden-local symmetric Lagrangian endowed with topology change and a univocal prediction on the deformation of the compact star, that could be measured in gravitational waves. The objective is not to offer a superior or improved EoS for compact stars but to confront with a forthcoming astrophysical observable the given model formulated in what is considered to be consistent with the premise of QCD. The model so obtained is found to satisfactorily describe the observation of a 2-solar mass neutron star^{1,2} with a minimum number of parameters. Specifically the observable we are considering in this paper is the tidal deformability parameter λ (equivalently the Love number k_2), which affects gravitational wave forms at the late period of inspiral stage. The forthcoming aLIGO and aVirgo observations of gravitational waves from binary neutron star system will provide a valuable guidance for arriving at a better understanding of highly compressed baryonic matter.

Keywords: compact star, equation of state, deformability, new scaling

1. INTRODUCTION

The observation of 2-solar mass neutron stars^{1,2} seems to indicate that the equation of state (EoS) for compact stars needs to be sufficiently stiffer to accommodate the mass larger than 1.5-solar mass. Moreover it requires the detailed information on the hadronic matter at higher density than the normal nuclear density, n_0 , which seems to be however much higher beyond the reach of presently planned terrestrial laboratories. After the recent detections of gravitational waves from binary black holes³, the expectation of detecting gravitational waves from a binary neutron stars and/or a black hole-neutron star binary becomes very optimistic than ever. The gravitational waves emitted during the binary inspiral phase up to merging can provide us the information on the dense hadronic matter of higher density at the core of compact stars.

The nuclei with large atomic numbers are already obvious examples

of highly dense matters composed of nucleons, $n \sim n_0 = 0.16/\text{fm}^{-3}$. The effective theories of nucleons have been developed and constrained by experimental data available up to and slightly above the normal nuclear density, n_0 . Hence they are fairly well controlled theoretically and experimentally. but the high density regime much above n_0 is more or less uncharted both experimentally and theoretically. Compact objects such as neutron stars are supposed to have higher core density than the normal nuclear density, n_0 . Roughly for neutron star with mass $\sim 1.5M_\odot$ the relevant density at the center is likely around $2n_0 - 3n_0$ and for the mass $\sim 2M_\odot$ the density is supposed to be larger than $\sim 5n_0$. On top of the possibilities of getting high density nuclear matter at terrestrial laboratories, for example FRIB(USA), FAIR(Germany), J-Parc(Japan) and RAON(Korea) in near future, the possible detections of gravitational waves from binary neutron stars(or binary neutron star black hole) are believed to be promising probes of the high density interior of neutron stars. Theoretically the success of low density effective theories univocally up to the normal nuclear density seems not to guarantee the similar success at higher density hadronic matter since the predictions of mass, radius, symmetry energy and deformability to name a few diverse from each other drastically beyond the normal nuclear density. In this sense we are now at the very exciting period of foreseeing the opportunity of constraining theories at higher density by experiments and observations.

For the highly dense hadronic matter, recently we proposed a new approach⁴⁻⁶ in which we try to formulate a field theory framework wherein both low and high density regimes are treated on the same footing. To cover both regimes in a consistent way in a unified field theoretic approach seems like a tall order. In this work, we discuss the physical properties of stellar matter using the newly proposed scheme of a new scaling law(BLPR) in medium^{7,8} all the way from normal nuclear density up to the higher density at the core of a neutron star. With a confrontation with the observed massive stars⁴, we discuss the physical properties of stellar matter with the EoS obtained therein. The relevant quantities are mass, radius and deformability parameters, which could be constrained by the gravitational wave forms emitted during the binary inspiraling phase. The aim here is then to confirm or falsify the strategies taken and assumptions made in Dong *et al.*⁴ and Lee *et al.*⁵ and also to find the directions to be taken in constructing the correct effective theory at higher density.

In section II, the basic concept of unified approach in this work is discussed. Using the minimal effective lagrangian in the frame work of

relativistic mean field, the equation of state of compact star with neutrons, protons, electrons and muon in weak equilibrium and charge neutrality condition is discussed in section III and the mass and radius are estimated. Tidal deformation with new stiffer EoS is discussed with the observational possibility in gravitational wave detections at aLIGO⁹ and aVirgo¹⁰ in section IV. The results are summarized in section V. We use units in which $c = G = 1$ and the notation in which Minkowski metric $\eta_{\mu\nu} = \text{diag}[-1, 1, 1, 1]$.

2. Unified approach with new scaling

The unified field theoretic approach is formulated using an effective theory which has sufficient number of degrees of freedom and parameters to be able to implement desired symmetries in high density regime or at the critical density, $n_c > n_0$. The first step is to construct the functional forms of parameters of the effective theory in terms of the variables in QCD, for example, quark and gluon condensates, by matching procedures near at QCD scale. Then, if we can compute the density dependence of QCD condensates, they automatically determine the density dependence of parameters in the effective theory. In addition the vacuum can be characterized by the expectation value of the relevant field, in this work scalar field $\langle \chi \rangle$, which is supposed to depend on the density. These are the intrinsic density dependencies(IDD) inherited from the quark/gluon condensates and scalar condensate. At low density (up to nuclear matter density) the success of chiral perturbation theory(ChPT) and low energy theorems implies that the QCD matching is implicitly taken care already. But at higher density, QCD matching or IDD will play a very crucial role since the naive extrapolation of ChPT, in which the QCD matching is implicit and therefore hidden, may go anywhere if not guided by the explicit QCD matching constraints.

The first step forward to that goal has been made using the effective lagrangian with minimum number of degree of freedom, which are the pseudo scalar mesons π , and scalar meson σ , vector mesons ρ and nucleon N . We adopt the relativistic mean field(RMF) approach, which are found to work remarkably well for finite and nuclear matter as well as for highly dense matter inside compact stars. The success of RMF can be understood that RMF captures the physics of the Landau Fermi liquid fixed point at which nuclear matter is located. It is assumed that the symmetries hidden at lower density on top of underlying chiral symmetry are the hidden local symmetry and the scale symmetry, which are supposed to be manifested explicitly

eventually at the critical density higher than normal nuclear density. The details are reviewed in recent articles^{5,6,11}.

When BR scaling(old-BR)¹² is applied to the neutron-star calculation using realistic NN potentials¹³, the mass is estimated to be in the range, $1.2M_{\odot} \sim 1.8M_{\odot}$, which is apparently less than $2M_{\odot}$. New BR/BLPR scaling has been proposed^{4,14} to incorporate the change in topology of the crystal structure of skyrmions, skyrmion \rightarrow half-skyrmion⁷. Suppose the threshold density, $n_{1/2}$, is higher but not so higher than the normal nuclear density, n_0 , then we expect the physical effect of such topology change on nuclear matter at the density $n \geq n_{1/2}$. More elaborated effective lagrangian including ω -meson has been discussed by Paeng *et al.*⁶ but in this work we use the simpler version discussed by Dong *et al.*⁴.

The drastic change in the symmetry energy observed at the density $n_{1/2}$ ^{7,8} can be translated into the parameter changes of the Lagrangian, a new scaling. With new scaling, new-BR/BLPR scaling, one can understand the origin of the drastic change in symmetry energy is due to the substantial change in tensor force, disappearance of ρ tensor component at higher density. The new scaling is incorporated into the V_{lowk} - implemented EFT approach to calculate the equation of state and the mass - radius relation of a compact object of a pure neutron matter. The stiffer equation of state has been obtained as expected and the mass can be as large as $2.4M_{\odot}$ ⁴, which seems to be consistent with recently observed high mass neutron stars. In this work we take a more realistic approach for the compact star with electrons, protons and neutrons, which are believed to be in weak equilibrium, rather than pure neutron matter. Near the surface of star, which is supposed to be in lower density region, $n < 0.5n_0$, we adopt the equation of state used in Hebeler *et al.*¹⁵.

It is assumed, in the range of density we are considering, the energy density of asymmetric nuclear matter($n_p \neq n_n$ or $n_p/n \neq 1/2$) can be described by the conventional form in terms of symmetry energy, $S(n)$, as given by

$$\epsilon_{nuc}(n, x) = \epsilon_{nuc}(n, x = 1/2) + n(1 - 2x)^2 S(n) \quad (1)$$

where $x \equiv n_p/n$ is the fraction of proton density. Then the symmetry energy factor $S(n)$ can be obtained by

$$S(n) = \epsilon_{nuc}(n, 0)/n - \epsilon_{nuc}(n, x = 1/2)/n \quad (2)$$

which is equivalent to the difference of binding energy per nucleon between the symmetric nuclear matter ($x = 1/2$) and the neutron matter ($x = 0$).

The pressure of nuclear matter is given by $p_{nuc} = n^2 \frac{\partial \epsilon(n)}{\partial n}$. In this work, we use the corresponding binding energy and the symmetry energy factor obtained by Dong *et al.*⁴ with the new scaling. In the weak equilibrium, the proton fraction, x , is determined essentially by the chemical potential difference between proton and neutron ,

$$\mu_n - \mu_p = 4(1 - x)S(n) \quad (3)$$

together with charge neutrality condition, $n_p = n_e + n_\mu$.

Now given EoS for energy density, ϵ , and pressure, p , the radius, R , and mass, $m(R)$, can be determined by integrating the Tolmann-Oppenheimer-Volkoff (TOV) equations^{16,17}. The equations are integrated up to the radius of the star, R , where $p(R) = 0$, and the mass of the star is determined by $m(R)$. The masses and radii, which depend on the equation of state, are important in predicting the gravitational waves emitted from the coalescing binary neutron stars. During the inspiral period of binary neutron stars, tidal distortions of neutron stars are expected and the resulting gravitational wave is expected to carry the corresponding information of equation of states¹⁸.

The tidal deformability of polytropic EoS, $p = K\epsilon^{1+1/n}$, where K is a pressure constant and n is the polytropic index, were evaluated by Flanagan and Hinderer^{19,20} and by others in more detail^{21,22}. Recent works by Reads *et al.*²³ and Hotokezaka *et al.*²⁴ demonstrate the measurability of the tidal deformation to constrain the EoS. In this work, we calculate the mass-radius relation and the tidal deformability using the stiffer EoS, which has been recently proposed with new scaling law(BLPR)^{4,6}.

3. Compact star composed of n, p, e, and μ

The asymmetry of neutron and proton numbers at high density, dictated by the chemical potential difference, inevitably leads to the weak equilibrium configuration with electron and muon with neutrinos escaped. It can be summarized by the relation between chemical potentials given by

$$\mu_n - \mu_p = \mu_e = \mu_\mu. \quad (4)$$

The chemical potential difference between neutron and proton should be the same as the electron chemical potential. The last equality is due to the muon emergence at higher density when the chemical potential difference from neutron and proton becomes larger than the muon mass. With the charge carriers, proton, electron and muon, the local charge neutrality

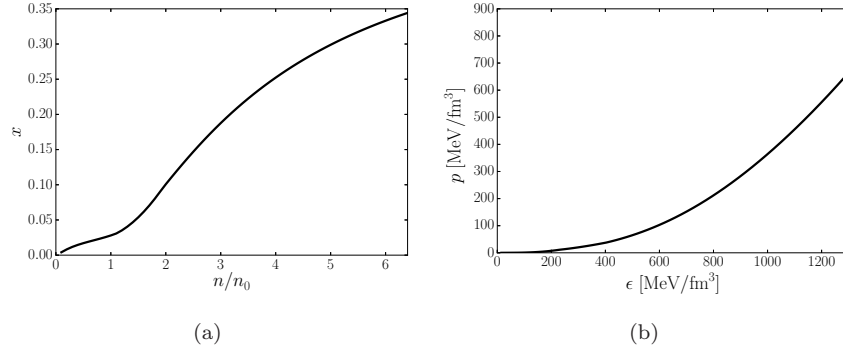


Fig. 1. (a) The proton ratio x as a function of nuclear density n . (b) The relation between energy density and pressure for an np asymmetric configuration.

condition is given by

$$n_p = n_e + n_\mu . \quad (5)$$

Using Eq. (3), Eq. (4), and Eq. (5) can be solved to get the carrier densities at a given density. The density dependence of proton fraction, x , is shown in Fig. 1(a). One can see that the proton fraction increases significantly as density increases.

The total energy density and pressure are given by

$$\epsilon(n, x) = \epsilon_{nuc} + \epsilon_{lep} , \quad (6)$$

$$p(n, x) = p_{nuc} + p_{lep} . \quad (7)$$

The energy density, ϵ_{lep} , and the pressure, p_{lep} are given by the degenerate fermi gas of the leptons (electron and muon) assuming a cold compact star ($T \sim 0$).

The resulting equation of state is shown as a pressure-energy density diagram in Fig. 1. The causality limit, $c_s \leq c$, constrains the highest density, n_c , beyond which the stiffer EoS used is no more valid. For EoS used in this work, it is found to be $n_c \sim 5.7 n_0$.

For a static and spherically symmetric astrophysical compact star, the metric is given by

$$ds^2 = -e^{\Phi(r)} dt^2 + e^{\Lambda(r)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 . \quad (8)$$

where Λ can be expressed in terms of a radial-dependent mass parameter,

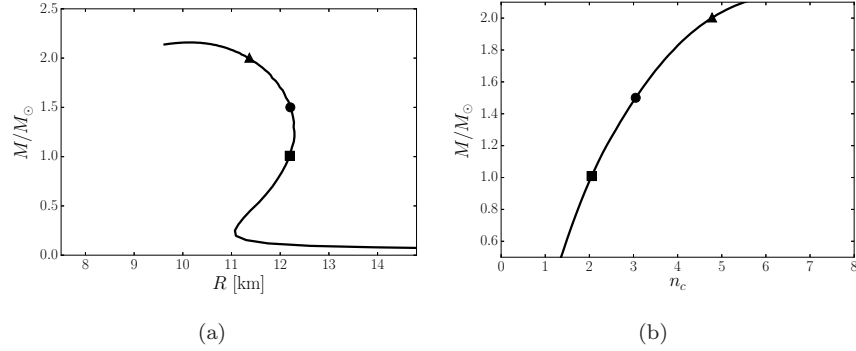


Fig. 2. (a) Mass(M)-Radius(R) curve. The filled-square, filled-circle, and filled-triangle correspond to $M = 1.0M_\odot$, $M = 1.5M_\odot$, and $M = 2.0M_\odot$ respectively. (b) n_c vs. Mass.

$m(r)$, introduced by

$$e^{\Lambda(r)} = \left(1 - \frac{2m}{r}\right)^{-1}. \quad (9)$$

Assuming a perfect-fluid stellar matter,

$$T_{\mu\nu} = (\epsilon + p)u_\mu u_\nu + pg_{\mu\nu}, \quad (10)$$

the relativistic hydrodynamic equilibrium is governed by the TOV equation

$$\frac{dm}{dr} = 4\pi r^2 \epsilon \quad (11)$$

$$\frac{dp}{dr} = -(\epsilon + p) \frac{m + 4\pi r^3 p}{r(r - 2m)} \quad (12)$$

$$\frac{d\Phi}{dr} = -\frac{1}{\epsilon + p} \frac{dp}{dr}. \quad (13)$$

where ϵ and p are energy density and pressure at r respectively and $u^\mu = dx^\mu/d\tau$ is the fluid's four-velocity. $m(r)$ is the mass enclosed inside the radius r . We can calculate the mass of compact star, M , and its radius, R , by integrating the TOV equation up to $p(R) = 0$ and we get the profile of $\Phi(r)$, $m(r)$ and $p(r)$.

The EoS of np asymmetric configuration is used to solve the TOV equation resulting in the mass-radius curve shown in Fig. 2. For np asymmetric configuration, the possible maximum mass is estimated to be $M \sim 2.1M_\odot$ with the radius $R \sim 11$ km, where the central density is about $5.7n_0$. For pure neutron matter⁴, the possible maximum mass is approximately

$M \sim 2.4M_\odot$ with the radius $R \sim 12\text{km}$ and $n \sim 4.7n_0$. In Fig. 2a, the filled-square, filled-circle and filled-triangle correspond to $M = 1.0M_\odot$, $M = 1.5M_\odot$, and $M = 2.0M_\odot$, respectively. The compactness $C = \frac{M}{R}$ in the range of mass $1.0 - 2M_\odot$ is found to be 0.12 - 0.26 and 0.14 for $1.4M_\odot$. Fig. 2b shows the relation between central density n_c and the radius and between n_c and the mass of the star.

4. Tidal Deformability

When a compact star is placed in a static external field, we suppose a star in a spherically symmetric configuration is then deformed by the external field. The asymptotic expansion of the metric at large distances r from the star defines the quadrupole moment, Q_{ij} , and the external tidal field, \mathcal{E}_{ij} , as expansion coefficients²⁵ given by

$$-\frac{1+g_{00}}{2} = -\left[\frac{m}{r} + \frac{3}{2} \frac{Q_{ij}}{r^3} n^i n^j + \dots \right] + \frac{1}{2} \mathcal{E}_{ij} r^2 n^i n^j + \dots, \quad (14)$$

where $n^i = x^i/r$ and Q_{ij} and \mathcal{E}_{ij} are both symmetric and traceless²⁵.

The deformability parameter λ is defined by

$$Q_{ij} = -\lambda \mathcal{E}_{ij}, \quad (15)$$

which depends on the EoS of the nuclear matter and provides the information how easily the star is deformed. The deformability parameter can be reexpressed by the dimensionless Love number, k_2 , defined by

$$\lambda = \frac{2k_2}{3} R^5. \quad (16)$$

In general, the linearized perturbations of the metric caused by an external field is given by²⁶,

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}, \quad (17)$$

where $g_{\mu\nu}^{(0)}$ is a unperturbed metric,

$$g_{\mu\nu}^{(0)} = \text{diag}[-e^{2\Phi(r)}, e^{2\Lambda(r)}, r^2, r^2 \sin^2 \theta] \quad (18)$$

where $\Phi(r)$ and $\Lambda(r)$ will be determined by the stress-energy tensor configuration discussed below. $h_{\mu\nu}$ is a linearized perturbation, which carries the information of Q_{ij} and \mathcal{E}_{ij} in Eq. (14). Since we will be considering the early stage of binary inspiral before the merging stage, the leading order tidal effects with even parity, $l = 2$, is dominated²⁷. In the Regge-Wheeler

gauge, the static and even-parity perturbation with $l = 2$ denoted by $H(r)$ and $K(r)$ can be written in the following form²⁰

$$h_{\mu\nu} = \text{diag} \left[-e^{2\Phi(r)} H(r) Y_{20}(\theta, \phi), \right. \\ \left. -e^{2\Lambda(r)} H(r) Y_{20}(\theta, \phi), \right. \\ \left. -r^2 K(r) Y_{20}(\theta, \phi), \right. \\ \left. -r^2 \sin^2 \theta K(r) Y_{20}(\theta, \phi) \right] . \quad (19)$$

$K(r)$ is related to $H(r)$,

$$K'(r) = H'(r) + 2H(r)\Phi'(r), \quad (20)$$

where the prime ' denotes the differentiation d/dr .

On the other hand, the non-vanishing component of the perturbation of stress-energy tensor, $\delta T_{\mu\nu}$, due to the tidal deformation, corresponding to $l = 2, m = 0$ metric perturbation are given by

$$\delta T_0^0 = -\delta\epsilon(r)Y_{20}, \quad \delta T_i^i = \delta p(r)Y_{20} . \quad (21)$$

Using the linearized Einstein equations,

$$\delta G_{\mu\nu} = 8\pi\delta T_{\mu\nu} , \quad (22)$$

where $G_{\mu\nu}$ is Einstein tensor, we obtain the differential equation for $H(r)$:

$$H'' + \left(\frac{2}{r} + \Phi' - \Lambda' \right) H' \\ + \left\{ 2(\Phi'' - \Phi'^2) - \frac{6}{r^2} e^{2\Lambda} + \frac{3}{r} \Lambda' \right. \\ \left. + \frac{7}{r} \Phi' - 2\Phi' \Lambda' + \frac{f}{r} (\Phi' + \Lambda') \right\} H = 0 , \quad (23)$$

where we introduce $f(r)$ for $\delta T_{\mu\nu}$, given by

$$f(r) = \frac{d\epsilon}{dp}. \quad (24)$$

At the asymptotic distance from the stellar matter, $r \gg M$, where $T_{\mu\nu} = 0$, g_{00} can be approximated

$$-\frac{1+g_{00}}{2} \rightarrow -\left[\frac{M}{r} - \frac{1}{2} H^{asympt}(r) Y_{20} \right], \quad (25)$$

where $H^{asympt}(r)$ is the solution of Eq. (23) at asymptotic distance given by

$$H^{asympt} = \frac{8}{5} \left(\frac{M}{r} \right)^3 c_1 + \left(\frac{r}{M} \right)^2 c_2 \cdots . \quad (26)$$

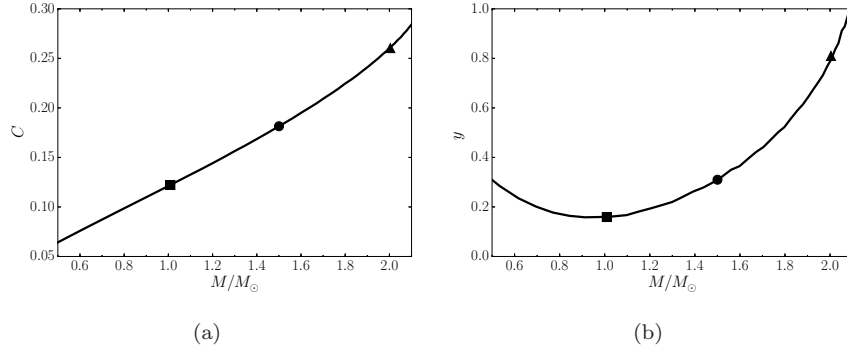


Fig. 3. (a) Compactness vs. Mass. (b) y vs. Mass.

One can determine c_1 and c_2 using the continuity of $H(r)$ and $H'(r)$ at the boundary, $r = R$, both for interior and outside solutions of Eq. (23). Then we can read out the deformability parameter by comparing Eq. (26) and Eq. (14),

$$\lambda = \frac{8}{15} M^5 \frac{c_1}{c_2}. \quad (27)$$

Using the continuity condition the $l = 2$ deformability λ can be written explicitly²⁰ in terms of the compactness $C = M/R$ and $y = RH'(R)/H(R)$,

$$\begin{aligned} \lambda_2 = & \frac{16}{15} R^5 C^5 (1 - 2C)^2 [2 + 2C(y - 1) - y] \\ & \times \left\{ 2C[6 - 3y + 3C(5y - 8)] \right. \\ & + 4C^3[13 - 11y + C(3y - 2) + 2C^2(1 + y)] \\ & + 3(1 - 2C)^2[2 - y + 2C(y - 1)] \\ & \left. \times \ln(1 - 2C) \right\}^{-1}. \end{aligned} \quad (28)$$

By solving TOV equation and Eq. (23) together, we can then calculate y and the compactness C (see Fig. 3) for the interior solution and we obtain λ or Love number, Eq. (28), as shown in Fig. 4. The compactness $C = M/R$ in the range of mass $1.4-2M_\odot$ is found to be 0.16 - 0.26 as shown in Fig. 3a.

The deformability parameter for $1.4M_\odot$ is found to be 2.86. It can be compared with those of different EoS's with only $npe\mu$ matter. For example, the EoS's of SLy²⁸, AP3²⁹ and MPA1³⁰ for the same mass give

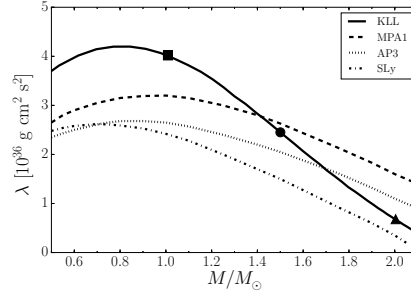


Fig. 4. The tidal deformability parameter λ in the mass range $0.5M_{\odot} - M_{max}$.

$\lambda = 1.70, 2.22$ and 2.79 respectively¹⁸. On the other hand, the slope of λ is found to be stiffer than those above in the mass range $1M_{\odot}-2M_{\odot}$. In the lower mass region around $\lesssim 1M_{\odot}$, the deformability parameter is found to be relatively higher than those of above EoS's ($\lambda < 3$), with the maximum value of 4.2 at $0.84M_{\odot}$. At higher mass region the deformability parameter in this work is lower than those EoS above mentioned.

It is interesting to note that recent numerical analysis^{23,24} demonstrated the measurability of tidal deformations* determined by the change of late inspiral wave forms for $\delta\Lambda > 400$.

5. Summary

We discussed the physical properties of stellar matter with a new stiffer EoS, which has been proposed recently using a new scaling law (new-BR/ BLPR) in medium caused by topology change at high density⁴, by extending Dong *et al.*'s work for pure neutron matter to a realistic nuclear matter of n , p , e and μ . The mass- radius and the tidal deformability were calculated.

The calculated maximum mass of compact star is found to be about $2M_{\odot}$ with its radius about 11km . The radius for the mass range of $1M_{\odot}-2M_{\odot}$ is found to be $11.2-12.2\text{km}$. The calculated deformability parameter for the stiffer EoS employed in this work is in the range $4.0-0.68$.

What characterizes the approach presented in this work is the stiffening of the EoS due to topology change predicted in the description of baryonic matter with skyrmions put on crystal background to access high density.

*In their analysis, the dimensionless form of deformability parameter, $\Lambda = G(c^2/GM)^5\lambda$ has been used. In this work $\Lambda = 634$ for $M = 1.35M_{\odot}$.

The change is implemented in the properties of the parameters of the effective Lagrangian anchored on chiral symmetry and manifests in nuclear EFT formulated in terms of RG-implemented V_{lowk} . Given that the approach describes fairly well the baryonic matter up to normal nuclear density, it is the changeover of skyrmions to half-skyrmions at a density $\sim (2-3)n_0$ that is distinctive of the model used. This topology change involves no change of symmetries – and hence no order parameters, therefore it does not belong to the conventional paradigm of phase transitions. But it impacts importantly on physical properties as described in various places in a way that is not present in standard nuclear physics approaches available in the literature. It is interesting to note that, as has been discussed recently^{31,32}, there is another way to produce the stiffening in EoS to access the massive compact stars. It is to implement a smooth changeover from hadronic matter – more or less well-described – to strongly correlated quark matter, typically described in NJL model. By tuning the parameters of the quark model so as to produce a changeover at a density $\gtrsim 2n_0$, it has been possible to reproduce the features compatible with the properties of observed massive stars.

After the detections of gravitational waves binary black holes, the possible detection of gravitational wave signals from coalescing binary neutron stars are well expected during the next run of aLIGO and aVirgo and the detection will inform us of the detailed effect of the tidal deformation^{18,19,23,24}. Recently the tidally modified waveforms have been developed up to the high frequency of merger^{23,33}, such that the deformability parameter λ , a function of the neutron-star EOS and mass, is measurable within the frequency range of the projected design sensitivity of aLIGO and aVirgo. It has been also demonstrated in Bayesian analysis that the tidal deformability can be measured to better than $\pm 1 \times 10^{36} \text{ g cm}^2 \text{ s}^2$ when multiple inspiral events from three detectors of aLIGO-aVirgo network^{34,35} are analyzed³⁶. They also show that the neutron star radius can be measured to better than ± 1 km. Thus the simultaneous measurement of mass, radius and deformability using gravitational wave detectors could present an exciting possibility to eventually pin down the highly uncertain EoS for the nuclear matter in the mass range of $1M_\odot$ – $2M_\odot$. This would provide a probe for the state of baryonic matter at the high density that is theoretically the most uncertain. And the hope is whether one can confirm or falsify the strategies taken and assumptions made in^{4,5} and whether the result would then help point the directions to be taken in the efforts described in^{4,6}.

Hyun Kyu Lee: “When Gerry invited me to Stony Brook in 1998 for my

sabbatical year, he put me in a house just next to his. One late afternoon he came to our place with a big smile and a basket of potatoes he just dug out in his yard. He had keen interest in hearing the news of detecting gravitational waves, which was one of his favorite laboratories up in the sky. I am now missing his big smile and a basket of comments on recent detections of gravitational waves, GW150914 and GW151226.”

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